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Mathematics Education & Some Methodological Considerations:

A brief survey with reference to Pupils in Katete District, Eastern province, Zambia

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Abstract

The research article seeks to determine the casual difference in performance in Circle Theorems when in concert with Concrete-Representational-Abstract (CRA) Instructional Approach. As a conventional method of instruction on Grade 11 pupils, the latter is compared with the traditional method of instruction in Mathematics education. In turn, Circle Theorems (CT) are viewed as properties that display relationships between angles within the geometry of a circle; and includes: Chord circle theorem, Tangent circle theorem, Cyclic quadrilateral circle theorem, Angle in a semi-circle theorem, Alternate segment circle theorem, Angle at the centre circle theorem and Angles in the segment circle theorem. The latter is not the main concern of this treatise. It conceptualises education as the process which does an all-round harmonious development of the individual to modify behaviour, attitude and thinking – all in its endeavour to investigate the effect of using Concrete–Representation–Abstract (CRA) Instructional Approach on Grade 11 pupils' performance in circle theorems in the Katete District, Eastern province, Zambia. It its methodology and design, it utilises descriptive statistics design; as the researchers used the difference-in-differences methods to assess the impact of the interventions on pupil performance. Overall, it establishes that there was a significant difference in scores between the pupils who were taught circle geometry using Concrete–Representational–Abstract (CRA) Instructional Approach and the pupils who were taught the same topic using the traditional approach for both post-test and delayed post-test (for post- test: Value= 65.667; sig= .004 < 0.05, for delayed post-test: Value = 78.333, sig= .000<0.05) according to means.

Keywords: Circle theorems, Concrete-representational-abstract, Instructional Approach, Traditional approach, Performance.

Introduction

Mathematics is considered one of the basic subjects of great importance in the various stages of education (Skovsmose, 2006). It contributes greatly to the development of students' mental abilities because of its direct or indirect application in daily life situations, which has earned it a prominent position among other study subjects. Learning of Mathematics is not an easy task as it is one of the fields that requires sequential growth within the different stages of growth and requires previous cognitive skills to learn new Mathematics concepts (Witzel, Mercer & Miller 2003). One of the most important components in Mathematics is the basic concepts of Mathematics (Salahat, 2022, kingdom of Saudi Arabia). The National Council of Teachers of Mathematics (NCTM, 2000) stressed the need for students to learn geometric concepts and identify the properties of geometric shapes because they develop their ability to distinguish between similarities and differences. Circle geometry as an aspect in Mathematics, plays an essential role in helping how mathematicians as well as learners of Mathematics appreciate and understand the space, shapes and orientations of various bodies and objects in this world. (Griffith, 2000). One aspect of geometry that enables mathematicians and learners of Mathematics appreciate circular space, shapes and orientation in this world is circle geometry, especially at the senior high school level which requires more innovation strategies that can enhance leaner's understanding and have skills in geometry.

Geometry is also one of the basic fields of school mathematics education. Students need a basic understanding of measurement and geometry at an early stage of learning to support their understanding on complex Mathematics later in life (Goldenberg & Clement, 2014). In addition, circle geometry provides a natural basis for developing reasoning and justification of skills (NCTM 2000), and helps connect facts, elicit results, build the students personality (Abu Zina, 2010) and provide opportunities to improve cognitive performance, communication processes and language understanding (Cawley et al, 2009). Moreover, circle geometry is one of the ten areas of knowledge related to professions based on Science, Technology, Engineering and Mathematics (STEM) (Carnevale et al 2011). Circle geometry is also interwoven with the individual's life and everything that surrounds them. This therefore, calls for increased interest in geometry, especially in the early stage of education where the focus is on circle geometric shapes, their properties and relationships among them and abstract thinking (Crompton, 2013). This study is therefore designed to find out the effect of Concrete-representational- abstract sequence of instruction approach on Grade 11 pupils performance in circle theorems. Other components are research objectives, research questions and significance of the research study.

Conceptual Clarifications

Education is a process which does all round harmonious development of the individual to modify behaviour, attitude and thinking. It also means training for the country and love for the nation. Engineering programmes are considered cognitively demanding and require at least very good Mathematics and science background at grade 12 O-Level examination results. Entry into these engineering programmes is based on one's marks achieved in Mathematics and Science in the final school leaving examinations certificate. The profile of engineering students differs strongly from that of students in other disciplines. An engineering student must be able to: apply knowledge of the Mathematics, Physics, and life sciences in order to understand, formulate and solve engineering problems; design and conduct experiments, analyse and interpret data; develop designs that meet specified requirements, design solutions to new problems and so on. Enrolling students with good academic performance in mathematics and science subjects is important as it allows higher learning institutions to identify those students who are most likely to complete their engineering studies successfully and on time. Enrolling under qualified students in the university constitutes a misuses of resources, share waste of time and above all under achievement of the education goals. Therefore, high scores in mathematics and science subjects at Grade 12 O-Level examinations would be the strongest predictors of academic performance in engineering.

One of the perennial problems of the education system in Zambia is high failure rate of students at Grade 12 examinations in Sciences and Mathematics subjects. This problem is as old as the education system in the country dating back to independence. Improving access to education has been also accorded a high priority in Zambia's education policies. Studies have shown that lack of education leads to high fertility rates, low life expectancy and high literacy rates which all affects national development (Wasanga 1997). Performance in Mathematics and Science subjects at O-Level certificate in many countries has been poor and Zambia inclusive. The importance of having a solid background in Mathematics is well recognised as it serves as a gateway to future professions in a variety of fields (Adedeji 2008, Pandor 2006). Mathematics is very important in our daily lives since it deals with real life situations in our daily activities (Ojose 2011).

A thorough understanding of Mathematics is an asset, if not essential for applicants interested to enrol in the school of Engineering. In other words, Mathematical competence is an essential component in preparing numerate citizens for employment and it is needed to ensure continued production of highly-skilled persons required by industry, science and technology (Steen 2001). According to Steen (2001), Mathematics does not only empower people with the capacity to control their lives but also provides science a firm foundation for effective theories; it also guarantees society a vigorous economy. Zambian Curriculum dating as back as early 1960s had incorporated sciences in the education system. These sciences comprised of general science at junior secondary and physics, biology, chemistry at senior level. Mathematics, science and technology are strongly influenced by global context and the proficiency in these disciplines is a pre-requisite to economic success. Numerous studies link the poor academic performance of learners to socio-economic problems, poverty, and politics (Engel Brecht et al. 1996). Therefore, this chapter discusses previous literature relevant to this study.

The initial study that culminated into this article was carried out with the realization that there were inadequate related local studies concerning Mathematics in Katete District of Eastern province of Zambia. In this research article, the focus was on the failure rate by the Grade 12 students in Mathematical concepts in circle geometry in Grade 12 examinations in Katete District Eastern Province and also, its impact on Mathematics achievements. Some of the reasons attributed to the poor achievement in Mathematics by scholars include; a shortage of qualified Mathematics teachers (Ohuche, 1989), poor facilities, equipment, and instructional materials for effective teaching, use of traditional chalk and talk methods, large pupils to teacher ratio and Mathematics phobia and fright, limited background preparation in Mathematics, lack of Mathematics teaching equipment and materials, fright and anxiety, low level of interest and some government policy, lack of problem-solving abilities, self-concept, and achievement motivation (Abu Zina 2010).

Mathematics is one of the oldest fields of study in the history of mankind. It is one of the most central components of human thought. It has been believed for centuries that Mathematics sharpens the human mind, develops logical thinking; and enhances reasoning abilities and spatial power. It influences an individual's personal development and contributes to the wealth of the country. This is mainly because it is at the heart of many successful careers and successful lives. The field of engineering is not an exception to this development. One of the chief aims of Mathematics has always been to reveal and describe an order in the natural world. If we look back to the early days of Mathematics, say four thousand to five thousand years ago, we will see what the Egyptian and Babylonian civilizations offered in Mathematics. They offered a very practical approach to Mathematics, answering questions that rarely extended beyond what was necessary to operate in daily life. During this time, rudimentary arithmetic and algebra were built up to answer questions in commerce and agriculture. The useful purposes for which they employed Mathematics dealt with money exchange, simple and compound interest, computing wages, expressing weights and lengths, dividing inheritances, and determining volumes of granaries and areas of fields. Their Mathematics was also used to study astronomy; making it possible to create calendars to accurately predict natural occurrences such as floods, something necessary for agricultural purposes. Accurate calendars could also be used for purposes of religious ceremonies, such as building temples so that the sun would shine on the altar at the appropriate time. These civilizations developed elementary arithmetic, notation, some early algebra, and basic empirical formulas in geometry. When considering classical Mathematics, the Greeks must be the main focus of our attention. The primary reason for discussing earlier Mathematics is to understand what the Greeks inherited and what they left to their posterity. Whereas the Egyptians and Babylonians produced a fairly crude and very practical Mathematics based on experience, the Greeks removed Mathematics from its practical underpinnings.

A major step in the advancement of Mathematics was the recognition that Mathematics in numbers and geometric figures can be dealt with in the abstract. This was not a small step in human thinking, and this initial step has been attributed to the Pythagorean School of Ancient Greece (Odili 1992). Cockcroft (1982) stated that there can be no doubt that every child should study Mathematics at school. He also highlighted that most people regard the study of Mathematics, together with that of science as being essential. For this reason, Mathematics is one of the core subjects in all schools worldwide as explained by the amount of time devoted to it in schools. In many countries, it is compulsory from primary to secondary levels of education. Colwell (2000) studied the performance of American students in the international Mathematics tests and noted that they were performing poorly in Kenya, the performance had been below average (Aoko 2015). Mathematics is an essential requirement by every field of academic aptitude and human growth to cope with the challenges of life and development. (Fajemidagba 1986, Akpan 1987) asserted that, Mathematics is the queen and servant of all school subjects, since it cuts across the school curricula. Therefore, Mathematics as a school subject affects all aspects of human life at different levels. For instance, Mathematics is significant in engineering, accounts, agriculture, Science and technology.

Internationally, Mathematics is one of the most influential subjects of all curriculum, and Mathematical understanding influences decision making in all areas of life, private, social and civil. Furthermore, Mathematics is used in counting, calculation and in measuring of various quantities of interest. Mathematics is applied at national level in the formulation of the national budget. There are countless famous people who have helped shape Mathematics. Many of the discoveries of these famous Mathematicians have roots in Science, medicine, and technology that are now common place. The application of Mathematics is so vast that it is used in almost every subject. We can that without Mathematics, then Science is not possible. In fact, when walking from one place to another place, our mind is also doing Mathematics, for instance to know the time at which we will arrive and spend. Almost every aspect of our day to day life involves Mathematics too. Mathematics learning is a must element in providing the child with the basic skills to live their life. It is one of the basic pillars for the child on which his/her life is, and would be standing. So the base of this pillar needs to be strong and clear. Therefore, the teaching and learning of Mathematics helps the child in developing analytical and reasoning skills with logical and structured thoughts. Mathematics is intimately connected to daily life and everybody's life- long planning. Shut out Mathematics from daily life and civilisation comes to a standstill. It is in the light of this background, that the researcher seeks to build and elicit among students and teachers the proper approach to teaching Mathematics concepts in circle geometry. The poor pass rate in Mathematics subject is a source of concern to the education system and the government as the economy of the country depend on how the citizens use the knowledge from this subject. Obe (1996) conceptualises Mathematics as the master and servant of most disciplines and thus, a source of enlightenment and understanding of the universe. He further opined that without it, the understanding of national problems would be superficial. Graeber and Weisman (1995) agreed that Mathematics helps the individual to understand the environment and to give accurate account of the physical phenomena around every person. Despite the important role that Mathematics plays in society, there has always been poor performance in the subject at national examinations (Aduda, 2003).

Mwamwenda (1995) argued that the achievement of students in a subject is determined by the approach of instruction rather than inability to study. Haimowitz (1989) indicated the cause of most failures in schools might not be due to insufficient or inadequate resources but to active resistance to use the instructional materials and approach such as Concrete-representational- abstract sequence of instruction. The argument suggests that favourable attitudes towards Mathematics should be developed for achievement in the subject to improve. Setidisho (2001) submitted that no other subject forms a strong binding force among various branches of Science as Mathematics, and without it, knowledge of the Sciences often remains superficial. Emphasising the importance of the subject to the society, Robert (1987) stated that in the United States, Mathematics has come to play important roles: in the engineering of highways, the search for energy, the designing of television sets, the profitable operation of most business, astronauts flying space-crafts, the study of epidemics, the navigation of ships at sea all depends on the study of mathematics.

The poor performance in Mathematics subject has been a continuing problem that necessitates concrete approach and strategy to address. It is necessary to do because Mathematics achievement can be considered as a measure of success in academic advancement. Despite the importance of Mathematics, it was found that many students face difficulties in learning, and teachers suffer in making students understand the subject (Hidayat, 2017). These low performing students cannot identify and carry out routine and obvious procedures, and they are incapable of employing basic algorithms, formulas, procedures or conventions to solve problems and interpret results literally.

The teacher's capacity is one of the important factors that contributes to the quality of education in the classroom. It is important for the teachers to modify and adjust their pedagogical skills to accelerate the learning process in the classroom (Behlol, Akbar, & Sehrish, 2018). Proper teaching methods characterize all good teaching, and priority may be given to improve the capacities and professional competencies of the teachers to apply appropriate teaching methods that may enhance the learning of students.

In trying to look at situations around the world, the Program for International Student Assessment (PISA) (2015) reported specifically in Mathematics achievement, among the seventy-two (72) participating countries, that only 24 or about only 33% performs above average while the 67% lies below the mean score. No less than 30 of the 56 other countries that participated in the Program for International Student Assessment (PISA) Math test had a larger percentage of students who scored at the international equivalent of the advanced level on our National Assessment of Educational Progress (NAEP) tests (Hanushek & Woessmann, 2008). Evidently, the poor quality of Mathematics education in South Africa can be traced to the low achievement levels reached by the students in standardized tests (Reddy et al., 2012). It was also found out that the African students in the international exams such as Trends in International Mathematics and Science Study (TIMSS) have consistently shown that students were not able to do less than the standardized and curricular expectations for their respective level (Sandefur, 2018).

Neighbouring countries in Asia like Singapore, Japan, Chinese Taipei, Vietnam, and Korea are among the Asian countries that have only reached the above-average, respectively (PISA, 2015). In Thailand, the result of the PISA test in 2015 shows that students had much lower scores than their neighbouring countries (Armstrong & Laksana, 2016). Thai students are underperforming their peers in several Asian countries as their scores were below the international average in Mathematic subject (Mala, 2016).

The fact that there is a continuing problem on the poor performance of students in Mathematics and the popularly acclaimed effectiveness of the Singaporean Concrete- Representational – Abstract Instructional Approach in helping students perform better in Mathematics is what prompted the researcher to conduct this study (Salingay & Tan, 2018). Specifically, the researcher would like to implement a quasi-experimental research design using the Concrete-Representational – Abstract Instructional Approach to determine whether it is applicable and effective for Zambian students. The limited studies showing how effective the Concrete- Representational – Abstract Instructional Approach is helping Filipino students learn Mathematics is another reason that prompted the researcher to conduct this study in Zambia.

During the pre-service and in – service teacher training, student teachers in various teacher training Institutions, would be teachers and indeed those seeking to upgrade their professional qualifications in various pedagogical skills and strategies are guided by methodology (Ellis, 2004). This is meant to prepare and produce teachers that can plan and deliver effective and impactful lessons in schools for learners to benefit both theoretically and practically. This can only occur if pupils grasp the concepts well and are later able to execute the principles and engage in thought provoking critical thinking, reflection and problem – solving (Campbell, 1993). Such a situation does not just arise from a vacuum in a classroom; it is a fruit of systematic and effective instruction and lesson facilitation in all subjects (CDC, 2013).

However, it can be reflected that the Zambian government has built infrastructure, put in resources, policies and strategies to facilitate the teaching and learning of Mathematics and sciences as its priority. To single out Mathematics, government and its local and international stakeholders have purposefully targeted access and quality in the teaching of Mathematics at all its levels (Lufeyo, 2021).This forms the nexus of this study to interrogate the effect of Concrete-Representational-Abstract (CRA) Instructional approach on Grade 11 pupils performance in circle theorems (geometry) in one selected secondary school in Katete District, Eastern Zambia. It is in this view that pre-existing literature has shown that the education system needs to be re- evaluated in Mathematical teaching practices in a manner that can boost students confidence in Mathematics (Kamina & Iyer, 2009).

Studies indicate that current teaching approaches in Mathematics, underestimate the amount of scaffolding practice, use of manipulatives and pictures students need to consolidate new concepts and skills (Adimora, Goldenberg & Clement, 2014). The teaching and learning of circle geometry has been of special interest to Mathematicians, educational stakeholders and scientist across the world (Mesa, Gomez & Cheah, 2012). The skills and knowledge learned from circle geometry help nurture students' ability to think critically and solve problems, such problems include; basic understanding of measurements and geometry at an early age to support their understanding of complex Mathematics later in life (Goldenberg & Clement, 2014). In addition, geometry provides a natural basis for developing reasoning and justification skills (NCTM, 2000), and helps connect facts, elicit results, build students' personality (Abu Zina, 2010), and provide opportunities to improve cognitive performance, communication processes, and language understanding (Cawley's, 2009). In addition, geometry represents the largest part of concrete Mathematics that is easy for the students to learn especially if the appropriate educational methods and manuals are available to understand and master.

Literature has confirmed that Concrete- Representational – Abstract Instructional Approach has evidently improved learner performance in learners when appropriately applied in other fields of mathematics such as probability, fractions, algebra, sets and many others. Furthermore, it is noteworthy that findings of enhanced interaction in lessons from the literature also encouraged the use of Concrete-representational- abstract sequence of instruction (Shadaan &Kwan. EU, 2013). Concrete-representational- abstract sequence of instruction is a learner- centred approach of teaching which in many fields of Mathematics has yielded positive results on pupil's performance (Kim, 2020).

Statement of the Problem

Examination results in mathematics have shown low percentage in performance in circle geometric questions. In the past ten years, the performance has been ranging from 34% to 46% recorded each time a test on circle theorems is given to grade 11 Pupils. The researchers assume that the challenge could be pedagogics where teachers are not applying the principles and methods of instruction embracing Concrete – Representation – Abstract (CRA) Instructional Approach. It is not yet known what the effects on pupils' performance could be if Concrete-representational-abstract sequence of instruction is employed than when not on Grade 11 pupils. It is assumed that there would be no difference on pupil performance in circle theorems when Grade 11 pupils are taught using Concrete-representational-abstract sequence of instruction compared to its counterpart method of traditional approach. In the light of the aforementioned problem, the researcher is empirically compelled to investigate the effect of using Concrete–Representation–Abstract (CRA) Instructional Approach on Grade 11 pupils' performance in circle theorems. Hence the need for a quasi –experimental research design.

Purpose of the research article

The purpose of the study that culminated into this research article was to determine the causal difference in performance in circle theorems when Concrete-representational-abstract sequence of instruction as a conventional method of instruction on Grade 11 pupils is compared to the traditional method of instruction in Mathematics education.

Research Objectives

This research article was designed to assess the existence of a causal difference in performance in circle theorems when Concrete-representational- abstract sequence of instruction as a conventional method of instruction on Grade 11 pupils as compared to the traditional method of instruction.

Specific Research Objectives

The initial research was guided by the following specific research objectives:

1). To assess the performance outcomes of Grade 11 pupils in Mathematics after using Concrete-representational-abstract sequence of instruction approach in the teaching of circle theorems at a secondary school in Katete District.

2). To assess the performance after using Traditional approach to Grade 11 pupils in teaching circle theorems at a secondary schools in Katete District.

3). To determine if there is any significant difference to account for causation in pupil performance in the achievement scores of Grade 11 pupils when circle theorems is taught using Concrete-representational- abstract sequence of instruction approach compared to its counterpart method of traditional approach.

Research Questions

The study was guided by the following research questions frame from the research objectives:

1). What is the performance outcome after using Concrete-representational- abstract sequence of instruction approach in the teaching of circle theorems in Mathematics to Grade 11 pupils in secondary schools in Katete District?

2). What is the performance outcome after using Traditional approach (TA) in the teaching of circle theorems in Mathematics to Grade 11 pupils in secondary schools in Katete District?

3). What difference was there in the performance in circle theorems of Grade 11 pupils taught using Concrete-representational- abstract sequence of instruction approach compared to its counterpart method of Traditional approach?

Methodology

Descriptive statistics on the study participants was presented as proportions for categorical variables and as mean values with corresponding standard deviation (SD) for continuous variables. When comparing the two groups, differences between proportions were assessed by chi-square test and differences between means with t-test or ANOVA as the case was. The conventional significance level of 0.05 was used in all analyses in order to reject the null hypothesis of no difference between groups. The researchers used the difference-in-differences methods to assess the impact of the interventions on pupil performance.

RESULTS

1.1 General Characteristics of Study Participants

The results of the findings of this research article, in which the researchers seeks to present, came from appendix one (1). These findings are from a sample of grade 11 pupils from two secondary schools whose ages were rather uniform ranging from16 to 18 years. Just over half n = 47 (52.2%) in the sample were males and less than half n = 43 (47.8%) were females (Figure 1.1).

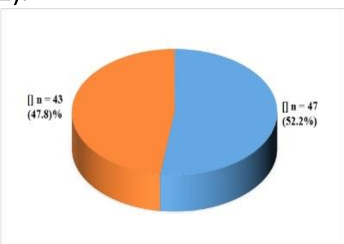


Figure 1.1: Distribution of respondents by gender

1.2 Performance Distribution of Control Group for Post Test and delayed Test

This section is related to research question two which is; "What is the performance outcome after utilising Concrete–Representational–Abstract (CRA) Instructional Approach in the teaching of circle theorems in Mathematics to Grade 11 pupils in secondary schools in Katete District?" This question ought to show the performances of both post-test and delayed post- test for a control Group.

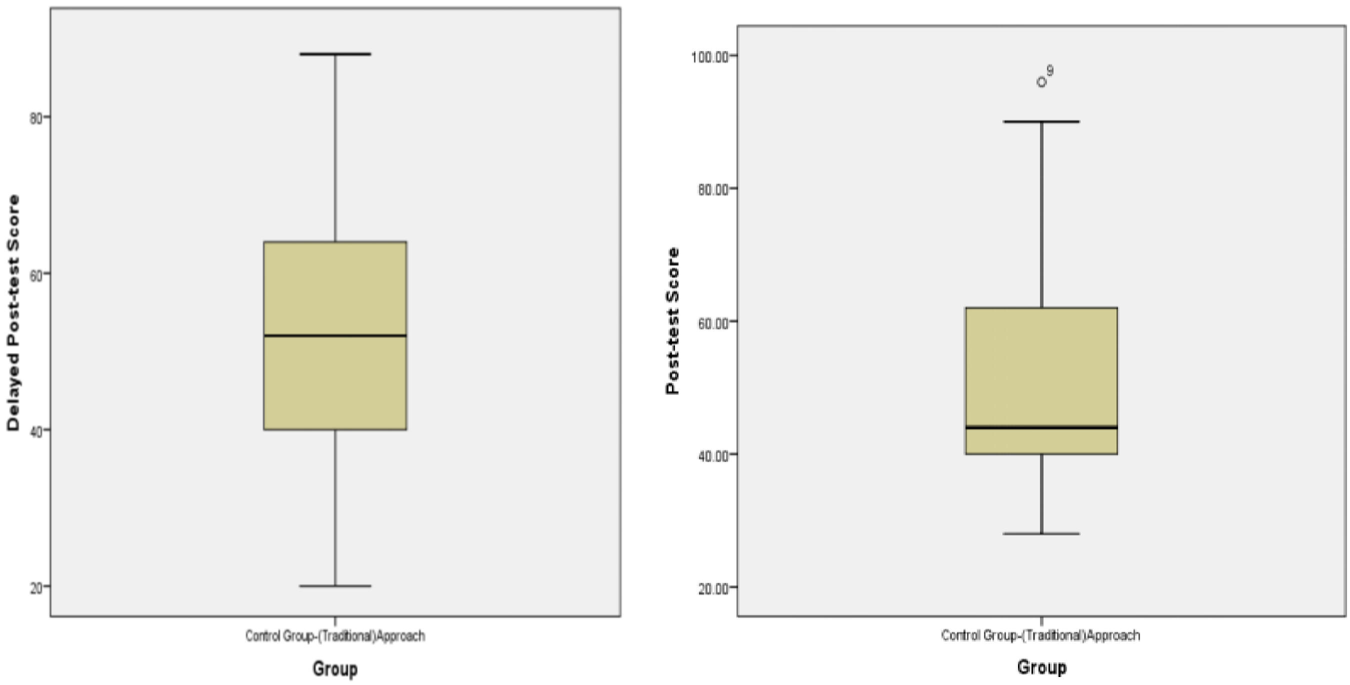
A performance distribution of the control group looking at the post-test and delayed post-test shows an unequal distribution of performance within the group. However, the distributions were non-normal as shown by the skewness values that are lower than 3 (See table 1.1). A statistical comparison of individuals within the control group was then performed. The within group assessment of differences was considered to be important in order to examine if the traditional method alone was effective. The researcher compares the test results after the intervention – traditional instruction and again four weeks after traditional approach, this traditional intervention compares the two test results. In this case, the researcher is not looking at the differences between two groups, but rather the differences between the same group taken at two time points' t_1 and t_2. The mean scores were rather close between the post-test (52.13%) and delayed post-tests (52.47%).

Table 1.1: Performance Distribution- Control Group for Post Test and delayed Test

Statistic	Post-test- Traditional Instructional Approach	Delayed Post Test- Traditional Instructional Approach
Mean	52.13	52.47
Median	44.00	52.00
Std. Deviation	17.30	18.30
Minimum	28.00	20.00
Maximum	96.00	88.00
Skewness	.969	.107
Kurtosis	.083	-.446

While the mean scores were rather close, the median scores showed a marked variation between the post-test (44) and the delayed post test scores (52). A pictorial presentation as shown in the box plot figure 1.2 below. It is evident that the remembrance scores following four weeks of the delayed post assessment were rather poor in the traditional approach.

(Left). Figure 1.2 Median scores Performance Distribution- Control Group for Post Test and delayed post test



The researcher assessed the distribution of grades with the control group and the findings show a mixed response. Though there was a rise in those who failed, students who were in the satisfactory group in the post-test moved into the merit and credit grades after four weeks as shown by the delayed post-test grades.

Figure 1.3 Grade Performance Distribution- Control Group for Post Test and Delayed Post-Test

The descriptive statistic in this section are suggestive of the existence of a difference between the post-test and delayed post-test in the group, a statistical comparison of individuals within this given group. This was one way of determining the effectiveness of the traditional intervention. Within group differences were considered by the researcher to be important. This called for the researcher to assess whether a difference existed just after the intervention of the program and again at the end of four weeks and compare the two at two time points t_i and t_2. One sample t tests below provide the data that the means are different. The traditional Instructional Approach's mean for the Post-test Score was 52.13 and for the Delayed Post Test-Approach was 52.47 for the control group as shown in Table 5.2. Table 5.3 shows that score was statistically significantly higher than a normal of 0 (the test level) t (44) = 20.20, p = .001.

Table 1.2 One-Sample Statistics -Control Group for Post Test and delayed Test

	N	Mean	Std. Deviation	Std. Error Mean
Group	45	2.00	.000[a]	.000
Post-test Score	45	52.13	17.3	2.57
Delayed Post-test Score	45	52.47	18.300	2.72

Table 1.3 One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Post-test Score	20.207	44	.000	52.13	46.93	57.33
Delayed Post-test Score	19.233	44	.000	52.47	46.97	57.96

1.3 Performance Distribution of the Experimental Group for Post Test and Delayed Test

This section is related to research question one which is; "What is the performance outcome after using Traditional approach (TA) in the teaching of circle theorems in Mathematics to Grade 11 pupils in secondary schools in Katete District?" This question ought to show the performances of both post-test and delayed post- test of an experimental Group. A performance distribution of the experimental group looking at the post test and delayed post-test shows an unequal distribution of performance within the group (see Table 1.4 and Figure 1.4).

Table 1.4: Performance Distribution Experimental Group for Post Test and Delayed Test

Statistic	Post-test- Abstract (CRA) Instructional Approach	Delayed Post Test- Abstract (CRA) Instructional Approach
Mean	83.87	85.58
Median	82.00	85.00
Std. Deviation	12.43	8.88

Minimum	53.00	58

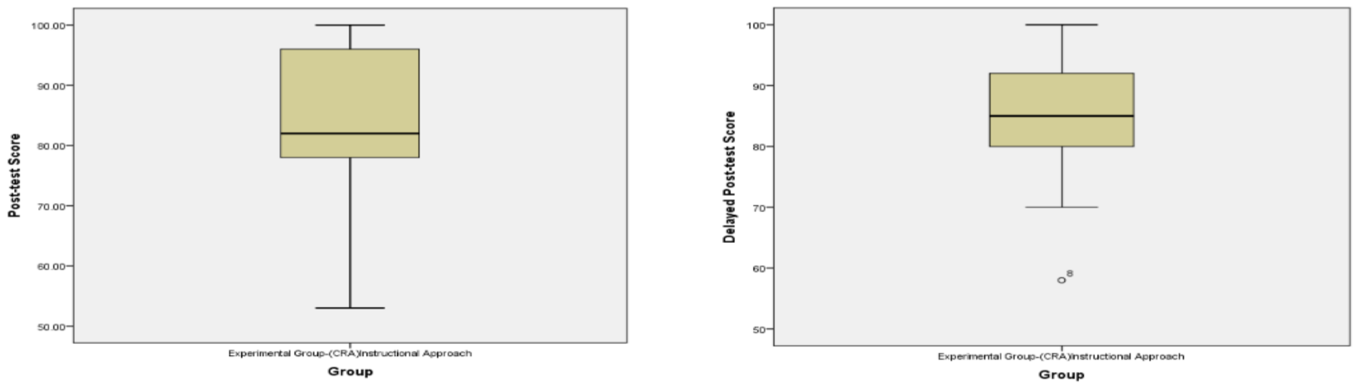


Figure 1.4 Median scores Performance Distribution Experimental Group for Post Test and delayed Test

The researcher assessed the distribution of grades with the experimental group and the findings show a mixed response. Though credit and merit scores were diminishing in the delayed post-test category, students were migrating into the distinction category after four weeks as shown by the delayed post-test grades. This seems to suggest greater pupil recall in the application of Abstract (CRA) Instruction Approach (see Figure 1.5).

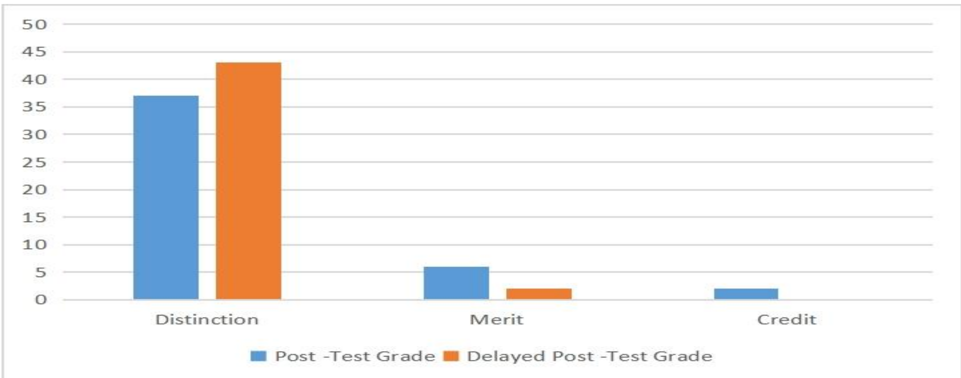


Figure 1.5 Grade Performance Distribution in the Experimental Group for Post Test and delayed Tests

The descriptive statistic in this section are suggestive of the existence of a difference between the post-test and delayed post-test in the group, a statistical comparison of individuals within this given group. This was one way of determining the effectiveness of Concrete-representational- abstract sequence of instruction approach intervention. Within group differences were considered by the researcher to be important. This called for the researcher to assess whether a difference existed just after the intervention of the program and again at the end of four weeks and compare the two at two time points t_i and t_2. One sample t tests below provide the data that the means are different. The Concrete-representational-abstract sequence of instruction approach’s mean for the Post-test Score was 83.8 and for the Delayed Post Test- Approach was 85.58 for the experimental group as shown in Table 1.5.

Table 1.5 One-Sample Statistics Experimental Group for Post Test and delayed Test

	N	Mean	Std. Deviation	Std. Error Mean
Group	45	1.00	.000a	.000
Post-test Score	45	83.86	12.42	1.85
Delayed Post-test Score	45	85.58	8.88	1.324

Table 5.6 shows that score was statistically significantly higher than a normal of 0 (the test level) $t(44) = 45.26$, $p = .001$.

Table 1.6 Experimental group One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Post-test Score	45.26	44	.000	83.86	80.13	87.60
Delayed Post-test Score	64.63	44	.000	85.57	82.91	88.25

Between Group Comparisons

This section is related to research question three which is; what difference is there in the performance in circle theorems of Grade 11 pupils taught using Concrete-representational- abstract sequence of instruction approach compared to its counterpart method of Traditional approach? This question ought to show the performances of both post-test and delayed post- test of a control and experimental Groups.

Between Groups differences examine how independent groups – groups that are not the same and in this case the control and experimental groups – may differ from each other on performance. Between Groups difference tests were done as they are useful for examining the efficacy of interventions or treatments. Each group is considered a single entity, and between-group comparisons were computed simultaneously. Since the data was not normally distributed, in order to determine the difference between the two groups, Mann-Whitney U test was employed.

Post-test Group Comparisons between Control and Experimental Groups

In Table 1.7, are Post test scores of the two groups and these show the actual significance value of the Mann-Whitney U-test. The Test Statistics table provides the test statistic, *U* statistic, as well as the asymptotic significance (2-tailed) *p*-value. The table concludes that the medians of the post test score are independent – and are statistically significantly higher that is 83.8 (for the experimental Group-(CRA) Instructional Approach (see Table 1.3) compared to 52.13 (See Table 1.6) for verification and this is shown as $U = 110$ and $p = .024$. We thus reject the null hypothesis which claimed that there is no difference between the post test scores in the two groups arising from the two interventions. There are indeed marked differences with experimental Group-(CRA) results higher than those of control group - (TA) in the post test scores.

Table 1.7 Post test scores Test Statistics

	Post test scores
Mann-Whitney	110.000
Wilcoxon W	320.000
Z	-2.446
Asymp.Sig. (2-tailed)	.024.
Exact Sig. [2*(1-tailed Sig.)]	.024[a]

a. Not corrected for ties
b. Grouping Variable: Group - Experimental Group-(CRA)Instructional Approach and Control Group-(Traditional)Approach

Delayed Post-test Group Comparisons between Control and Experimental Group

In Table 1.8, displayed are Delayed Post test scores of the two groups and these show the actual significance value of the Mann-Whitney U test. The table concludes that the medians of the delayed post test score are independent – and are statistically significantly higher that is 85.5 (for the experimental Group-(CRA) Instructional Approach (see Table 1.3) compared to 52.47 (See Table 1.1) and for verification, this is shown as $U = 221$ and $p = .001$ in Table 1.8 below. We thus reject the null hypothesis which claimed that there is no difference between the post test scores in the two groups arising from the two interventions. There are indeed marked differences with experimental Group-(CRA) results higher than those of control group - (TA) in the delayed post test scores.

Table 1.8: Delayed Post test scores Test Statistics

	Post test scores
Mann-Whitney	221.000
Wilcoxon W	465.000
Z	-1.722
Asymp.Sig. (2-tailed)	.001.
Exact Sig. [2*(1-tailed Sig.)]	.001[a]

a. Not corrected for ties

b. Grouping Variable: Group - Experimental Group-(CRA)Instructional Approach and Control Group-(Traditional)Approach

Summary of Mean improvement by groups

As indicated above a repeated measures ANOVA summarised below was performed to answer the research question “*What difference is there in the performance in circle theorems of Grade 11 pupils taught using Concrete-representational- abstract sequence of instruction approach compared to its counterpart method of Traditional approach?”*

With this question, one- way ANOVA was performed (see (see Tables 1.8a and 1.8b) to determine if there were no significant differences between CRA treatment and control groups.

Table 1.8a ANOVA (Descriptive)

	Mean	Std deviation	Std Error	95% Confidence interval for Mean Lower Bound	Upper Bound	Minimum	Maximum
Experimental (CRA)	83.87	12.429	1.853	80.13	87.60	53	100
Post- test (%)							
Control (Traditional)	52.13	17.307	2.580	46.93	57.33	28	96
Total	68.00	21.887	2.307	63.42	72.58	28	100
Experimental (CRA)	85.58	8.882	1.324	82.91	88.25	58	100
Delayed Post- test (%)							
Control (Traditional)	52.47	18.300	2.728	46.97	57.96	20	88
Total							
	69.02	21.948	2.314	64.43	73.62	20	100

Table 1.8b: Results of One- Way ANOVA

	Sum of Squares	df	Mean Squares	F	sig
Between Groups	22657.600	1	22657.600	99.811	
Post- test (%)	19976.400	88	227.005		.000
Within Groups	42634.000	89			
Total					
Between Groups	24667.778	1	24667.778	119.232	
Delayed Post- test (%Within	18206.178	88	206.888		.000
Groups **Total**	42873.956	89			

To validate the results, the researcher performed a one-way Analysis of Variance (ANOVA) using the SPSS version 23.0. One- way ANOVA test was applied to observe whether there was a significant difference in performance between the two groups (a group taught using Concrete–Representational – Abstract (CRA) Instructional Approach and the other one taught using traditional approach). Results presented in tables (1.8a and 1.8b) showed a significant main effect for instruction based on simple comparisons from the post- test and the delayed post- test assessment.

According to the results of the analysis, there was a significant difference in scores between the pupils who were taught circle geometry using Concrete – Representational – Abstract (CRA) Instructional Approach and the pupils who were taught the same topic using the traditional approach for both post- test and delayed post- test (for post- test: F= 99.811; p= .000 < 0.05, for delayed post- test: F= 119.232, p= .000<0.05) according to means.

In trying to investigate further on the effect of Concrete- Representational- Abstract Instructional Approach on Grade 11 pupils' performance in circle theorems and the difference in performance of pupils taught using Concrete – Representational – Abstract (CRA) Instructional against the counterpart method of Traditional approach, the researcher also performed the Chi Square analysis on the following hypotheses:

H_0=There is no significant mean difference in the performance in post-test and delayed post-test between the experimental group taught with Concrete – Representational – Abstract (CRA) Instructional Approach and the control group taught with traditional method.

Table 1.9a: Post- test Analysis Using the Chi Square

	Value	df	Asymptotic significance (2- sided)
Pearson chi- Square	65.667	38	.004
Likelihood Ratio	89.541	38	.000
Linear- by- Linear Association	47.299	1	.000
No. Valid Cases	90		

Table 1.9b: delayed Post- test Analysis Using the Chi Square

	Value	df	Asymptotic significance (2- sided)
Pearson chi- Square	78.333	39	.000
Likelihood Ratio	106.682	39	.000
Linear- by- Linear Association	51.207	1	.000
No. Valid Cases	90		

Tables 1.9a and 1.9.babove compared the performance of two groups (treatment and control) using the Chi- Square analysis for post- test and delayed post-test.

According to the results of the analysis, there was a significant difference in scores between the pupils who were taught circle geometry using Concrete–Representational–Abstract (CRA) Instructional Approach and the pupils who were taught the same topic using the traditional approach for both post- test and delayed post- test (for post- test: Value= 65.667; sig= .004 < 0.05, for delayed post- test: Value = 78.333, sig= .000<0.05) according to means. Therefore, we fail to reject and conclude that there is significant difference in performance between the group which was taught circle geometry using CRA instruction approach and the group which was taught using the tradition approach.

Conclusion

The two group performance scores showed differences in post-test and delayed post test results. The delayed post-test performance scores were higher than the post test scores in either case. However, generally, TA control group scores were lower than those of the experimental CRA group.

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